Abstract: - This paper deals with the sliding mode control adjustment of a speed control for DC motor. Firstly, the paper introduces the principle of sliding mode control method. Then, design controller for DC motor after that the comparison between PID and Fuzzy is made on the real model of the DC motor. The main result of the paper is the analysis the terminal sliding mode control. After obtaining the entire model of speed control system, the model is utilized with MATLAB (SIMULINK).

Key-Words: DC motor; sliding mode control; speed control; MATLAB (SIMULINK).

1 Introduction

DC motors are widely used in robotic and industrial equipment where high accuracy is needed. In some cases the uncertain conditions encounter the DC motor control to some difficulties. Hence, DC motor control has been stimulated a great deal of interest from several decades ago up to now [1] DC motors are identified as adjustable speed machines for many years and a wide range of options have evolved for this purpose. D.C motor is Considered as a SISO (Single Input and Single Output) system which has speed characteristics and is compatible with most mechanical loads. By proper adjustment of the terminal voltage [3] the mentioned characteristic makes a D.C motor controllable over a wide range of speeds. In this article speed of DC motor control by using sliding mode controller.

2 Sliding mode control

A linear system can be described in the state space as follows [12]:

\[
\dot{x} = Ax + Bu
\]  

(1)

Where \(x \in R^n\), \(u \in R\), \(A \in R^{n\times n}\) and \(B \in R^n\) and B is full rank matrix. A and B are controllable matrixes. The functions of state variables are known as switching function:

\[
\sigma = Sx
\]  

(2)

The main idea in sliding mode control are[1]:

- Designing the switching function so that \(\sigma = 0\) manifold (sliding mode) provide the desired dynamic.
- Finding a controller ensuring sliding mode of the system occurs in finite time
First of all, the system should be converted to its regular form:

\[ \dot{x} = Tx \]  

(3)

T is the matrix that brings the system to its regular form:

\[
\begin{align*}
\dot{x}_1 &= \tilde{A}_{11}x_1 + \tilde{A}_{12}x_2 \\
\dot{x}_2 &= \tilde{A}_{21}x_1 + \tilde{A}_{22}x_2 + B_2u
\end{align*}
\]  

(4)

The switching function in regular form is:

\[ \sigma = \bar{S}_1\dot{x}_1 + \bar{S}_2\dot{x}_2 \]  

(5)

On the sliding mode manifold ( \( \sigma = 0 \)):

\[ \bar{x}_2 = -\bar{S}_2^{-1}\bar{S}_1\dot{x}_1 \]  

(6)

By using (6) And (4):

\[ \dot{x}_1 = (\tilde{A}_{11} - \tilde{A}_{12}\bar{S}_2^{-1}\bar{S}_1)\bar{x}_1 \]  

(7)

One of matrixes in product \( \bar{S}_2^{-1}\bar{S}_1 \) should be chosen arbitrary. Usually (8) is used to ensure that \( \bar{S}_2 \) is invertible:

\[ \bar{S}_2 = B_2^{-1} \]  

(8)

\( \bar{S}_1 \) can be calculated by assigning the Eigen value of (7) by pole placement method. Hence, switching function will be obtained as follows:

\[ S = [\bar{S}_1 \bar{S}_2]^T \]  

(9)

The control rule is:

\[ u = u_c + u_d \]  

(10)

Where \( u_c \) and \( u_d \) are continuous and discrete parts, respectively and can be calculated as follows:

\[ u_c = -\tilde{A}_{21}\dot{x}_1 - \tilde{A}_{22}\sigma \]  

(11)

\[ u_d = -K_S \text{sgn}\sigma - K_P\sigma \]  

(12)

Where sgn is sign function. \( K_S \) and \( K_P \) are constants calculated regarding to lyapunov stability function.

3 Modeling of DC motor

The state space model of DC motor is as follows [1]:

\[ \dot{x} = Ax + Bu = \begin{bmatrix} -\frac{b}{J} & \frac{km}{L} \\ -\frac{J}{L} & -\frac{R}{L} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]  

(14)

In this equation \( x \) is two dimensional vector \( x = [x_1, x_2] \) where \( x_1 \) and \( x_2 \) are angular velocity of shaft and armature current, respectively. \( u \) is the armature voltage. \( R \) and \( L \) are the resistance and inductance of the armature coil. \( k_e \), \( b \), \( J \), and \( k_m \) are velocity constant, viscous friction, moment inertia and torque constant, respectively.

By using the Laplace transform of (14), the transfer functions of system according to angular speed of shaft \( (W(s)) \) and armature voltage \( (U(s)) \) can be calculated:
\[
W(s) = \frac{k_m}{s^2 + \left(\frac{R}{L} + \frac{b}{J}\right)s + \left(\frac{Rb + k_m k}{JL}\right)}
\]  

Equation (15) in time domain is as follows:

\[
\frac{d^2\omega}{dt^2} + \left(\frac{R}{L} + \frac{b}{J}\right)\frac{d\omega}{dt} + \left(\frac{Rb + k_m k}{JL}\right)\omega = \frac{k_m}{JL}u
\]  

(16)

However, if the state variables consider \( \omega \) and \( \dot{\omega} \). The system described by equation (14) by equation (17) will be expressed, Where the only variable is the angular velocity and derivative.

\[
\dot{x} = \begin{bmatrix} 0 & 1 \\ A_1 & A_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ k_m \end{bmatrix} u
\]  

(17)

The relation (17) \( A_1 \) and \( A_2 \) are obtained as follows

\[
A_1 = -\left(\frac{Rb + k_m k}{JL}\right)
\]  

(18)

\[
A_2 = -\left(\frac{R}{L} + \frac{b}{J}\right)
\]  

(19)

**A. Design of the switching function**

We are going to set the angular velocity over a certain value \( r \), so switching function is

\[
\sigma = s_1 (\bar{x}_1 - r) + s_2 \bar{x}_2
\]  

(20)

If the controller switching function is designed to be placed on the surface \( \sigma = 0 \) then Solving equations (20) assume \( \sigma = 0 \), \( \omega \) and \( \dot{\omega} \) are obtained by

\[
\omega = r \left(1 - e^{-\frac{s_1}{s_2}}\right)
\]  

(21)

\[
\dot{\omega} = r \frac{s_1}{s_2} e^{-\frac{s_1}{s_2}}
\]  

(22)

As equation (17) it is regular form, so the transformation matrix is equal to the unit matrix Factor \( s_2 \) according to equation (8) must be calculated

\[
s_2 = \frac{JL}{k_m}
\]  

(23)

Also according to (1-8) \( s_1 \), calculated and \( w \) Pole placement method using (1-10). Suppose we have to placed system poles in \( \lambda \) so we have

\[
\frac{s_1}{s_2} = -\lambda
\]  

(24)

As (21), (22) and (24) shown \( \lambda \) determines the speed of convergence of the system output. So it is better to choose a small negative value. Thus, the switching function was designed as follows.

\[
\sigma = s_2 \left(-\lambda (\bar{x}_1 - r) + \bar{x}_2\right) = \frac{JL}{k_m} \left(-\lambda (\omega - r) + \dot{\omega}\right)
\]  

(25)

**B. Controller design**

If the equation (17) can be rewritten based on the state variables \( \sigma \) and \( x_1 = (\bar{x}_1 - r) \). The following is reached

\[
\begin{bmatrix} \dot{X}_1 \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n
\]  

(26)
That (26) has the following parameters and variables.

\[ \tilde{A}_{11} = -\frac{s_1}{s_2} = \lambda \]  
(27)

\[ \tilde{A}_{12} = \frac{1}{s_2} \]  
(28)

\[ \tilde{A}_{21} = A_1 s_2 - A_2 s_1 - \frac{s_1^2}{s_2} = s_2 (A_1 + A_2 \lambda - \lambda^2) \]  
(29)

\[ \tilde{A}_{22} = A_2 + \frac{s_1}{s_2} = A_2 - \lambda \]  
(30)

\[ u_a = s_2^{-1} u + A_1 r \]  
(31)

Thus the relations (10), (11) and (12) controller for the system (26) is designed as follows.

\[ u_a = -\tilde{A}_{11} X 1 - \tilde{A}_{22} \sigma - K_j sgn(\sigma) - K_p \sigma \]  
(32)

(33) Sets armature voltage feedback based on the derivative of the angular velocity for motor

\[ u = -s_2 [A_1 r + s_2 (A_1 + A_2 \lambda - \lambda^2) \omega - r] + (A_2 - \lambda) \sigma + K_j sgn(\sigma) + K_p \sigma \]  
(33)

If \( A_1 r = A_4 \omega - A_1 (\omega - r) \) in (33) then we have

\[ u = -s_2 \{ A_1 \omega + [s_2 (A_1 + A_2 \lambda - \lambda^2) \\
- A_1 ](\omega - r) + (A_2 - \lambda + K_p) \sigma + K_j sgn(\sigma) \} \]  
(34)

So the sliding mode controller is:

\[ u = \left( \frac{J L}{k_a} \right) \left( \frac{R k_a + k_m}{J L} \right) \omega + \left[ \left( \frac{J L}{k_a} \right) \left( \frac{R k_a + k_m}{J L} \right) + \left( \frac{R}{L} + \frac{b}{J} \right) \lambda + \lambda^2 \right] \]  

\[ - \left( \frac{R k_a + k_m}{J L} \right) \omega - \left( \frac{R}{L} + \frac{b}{J} \right) \lambda - K_p \sigma - K_j sgn(\sigma) \]  
(35)

\[ \sigma = 0.09157 \times 10^{-4} (\omega - r) + 0.09157 \times 10^{-6} \dot{\omega} \]  
(36)

\[ u = (0.09157 \times 10^{-6}) (3675896.1 \omega - 3675895.83 (\omega - r) + 7491.2567 \sigma - sgn(\sigma)) \]  
(37)

Figure 1 scheme of the controller equation (36) shows.

C. Switching function and controller design for a real motor

Switching function of sliding mode controller for DC motor control method according to the relations (35) and (33) are designed.

If the motor parameters like table (1), then the controller we will numerically designed as follows.

4 MATLAB simulation results
To evaluate the performance of the sliding mode control (SMC), it is compared with a PID controller, and fuzzy controller. A maxon motor is used in all experiments. The parameter of this motor is shown in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$7.17 \Omega$</td>
</tr>
<tr>
<td>$L$</td>
<td>$0.953 \times 10^{-3} , H$</td>
</tr>
<tr>
<td>$k_e$</td>
<td>$0.29 V/s$</td>
</tr>
<tr>
<td>$k_m$</td>
<td>$46 \times 10^{-3} , NmA^{-1}$</td>
</tr>
<tr>
<td>$J$</td>
<td>$4.42 \times 10^{-6} , Kgm^2$</td>
</tr>
<tr>
<td>$b$</td>
<td>$2.99 \times 10^{-4} , Nms$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$-100$</td>
</tr>
<tr>
<td>$K_s$</td>
<td>1</td>
</tr>
<tr>
<td>$K_p$</td>
<td>0</td>
</tr>
</tbody>
</table>

Block diagram of the sliding mode controller is implemented with the relations (36) and (37) in the Matlab SIMULINK (Figure 2) is displayed.

Figure 3: the output scope of figure 2

Figure 4 shows the angular velocity of DC motor with respect to time. As can be seen in this figure, the PID controller despite to good settling time causes sever overshoot.

Figure 4: PID output

As this figure (5) confirms the steady state fluctuations of fuzzy controller is much more than others. In comparison with PID controller.

Figure 2: DC motor model sliding mode controller is implemented in SIMULINK
5 Conclusion

In this paper sliding mode control (SMC) proposed to speed control of DC motor. At first for controlling speed of DC motor a simplified closed loop is utilized. Then DC motor is modeled after that speed controller is designed.

As sliding mode control is based on the system Dynamic characteristics also it took a lack of influence of external disturbances from user as result it worked more useful and results confirms that used sliding mode control for speed control is more efficient in comparison with fuzzy and PID controllers.

References:

[7] Infineon Technologies, Basic DC motor speed PID control with the Infineon Technologies